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# UNIVERSITY OF WICHITA

ON THE LIFT OF A BLOWING WING IN A PARALLEL STREAM

by H.B. Helmbold

Engineering Study No. 110

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August 1953
University of Wichita
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### 1. General relationships.

When a jet is issued through a blowing slot in the downstream direction tangentially to the surface or at the trailing-edge of the wing an additional lift will result. Let $\rho$ denote the air density, b the span of wing and jet, h the thickness of the jet at the trailing edge, V the undisturbed velocity, v the excess velocity in the jet, $w_j$ the downwash velocity of the jet at infinity and w the downwash velocity of the flow adjacent to the jet at infinity. Furthermore assume that no mixing occurs between jet and surrounding flow, that thicknesses are small in comparison with the span and chord dimensions of the wing and do not vary appreciably from trailing edge to infinity (approximately constant pressure), and that the downwash angle at infinity is spanwise constant and small, i.e.

$$\frac{w_j}{V+v} = \frac{w}{V} \quad . \tag{1}$$

Then the lift will be equal to the change of downward momentum flux imparted by the co-operation of jet and wing

$$L = \rho bh(V+v)w_j + \rho \cdot \frac{\pi}{4}b^2 V \cdot w \tag{2}$$

or, by eq. 1,

$$L = \rho bh(V+v)w_j + \rho \cdot \frac{\pi}{4}b^2 \frac{V^2}{V+v} w_j = L_I + L_C \tag{3}$$

The first term on the right-hand side ($L_I$) represents the downward momentum flux of the jet itself and the second term ($L_C$) represents the lift produced by the wing under the influence of the jet. Physically, the two components can be distinguished by the fact that the first term implies only transversal vortices on the jet surface whereas the second term implies only longitudinal vortices on the jet surface which must be closed by the circulation of the wing. Therefore the terms $L_I$ and $L_C$ may be called momentum lift and circulation lift, respectively.

When the velocity V of the basic flow vanishes, the lift produced by the jet is purely momentum lift, in the above sense, viz.

$$L = L_I = \rho bh v w_j = \rho bh v^2 \sin \delta; \quad L_C = 0 \tag{4}$$

since $w_j = v \sin \delta$, where $\delta$ denotes the angle between the jet and the vanishing velocity V.

The downwash velocity of the jet at infinity is, by eq. 1,

$$w_j = \frac{L}{\rho b \left[ h(V+v) + \frac{\pi}{4} b \frac{V^2}{V+v} \right]} . \tag{5}$$

When $b \to \infty$ ,

$$w_j \to \frac{L}{\rho \cdot \frac{\pi}{4} b^2 \frac{V^2}{V+v}} \to 0, \text{ if } \frac{L}{b} \text{ and } V \text{ are finite.} \tag{6}$$

This means that in the two-dimensional flow the jet finally is turned in the direction of the undisturbed velocity V.

The ratio of momentum lift to circulation lift is

$$\frac{L_I}{L_C} = \frac{4}{\pi} \frac{h}{b} \left(1 + \frac{v}{V}\right)^2 . \tag{7}$$

when $b \to \infty$ ,

$$\frac{L_I}{L_C} \to 0; \quad L \to L_C \tag{8}$$

and $\quad \frac{L_I}{b} \to 0$, if $\frac{L}{b}$ is finite. $\tag{8a}$

In other words, the two-dimensional-flow lift is purely circulation lift, according to the above sense.

## 2. The two-dimensional flow around a blowing wing.

Experience shows that the additional lift of a blowing wing is considerably higher, say 5 times, than the vertical component

$$I_y = \rho b h (V+v)^2 \sin \delta_{TE}$$

of the momentum flux of the jet at the trailing edge. This fact in itself says that the additional lift must be essentially a circulation lift. As can be concluded from the above considerations a circulation must be produced by the turning action of the basic flow on the jet. This will now be studied in detail.

The pressure gradient $\partial p/\partial n$ normal to a curved streamline is in equilibrium with the centrifugal acceleration times the density $\rho$ or,

$$\frac{\partial n}{\partial n} = \rho \frac{q^2}{r} ,$$

where q denotes the local velocity and r the radius of curvature. Integrating across the thin sheet of jet we get as a first approximation for the pressure increase across the jet

$$\Delta p_j = \rho(V+v)^2 \frac{h}{r} . \tag{9}$$

Because the curvature is identical for the jet and the adjacent basic flow it makes sense to compare this pressure increase with the pressure increase across a thin sheet of the basic flow with the same geometrical shape. Neglecting the velocity variation in the basic flow (q = V) we get as a first approximation to the pressure increase across an identical streamtube of the basic flow

$$\Delta p_a = \rho V^2 \frac{h}{r} . \tag{10}$$

Since really in the streamtube under consideration the jet is moving and not the basic flow, an additional pressure increase is produced by the greater centrifugal acceleration of the jet

$$\Delta p_j - \Delta p_a = \rho \left[ (V+v)^2 - V^2 \right] \frac{h}{r} . \tag{11}$$
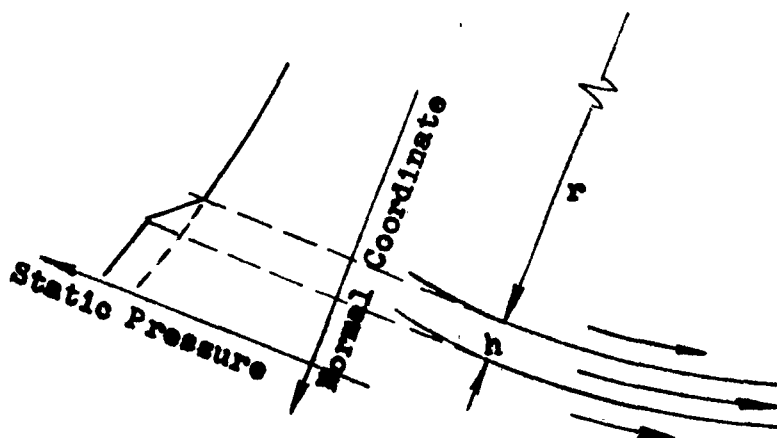


Figure 1

In other words, the jet supports a pressure discontinuity in the basic flow (Fig. 1). If $V_1$ and $V_2$ denote the basic-flow velocities at opposite points of the jet boundaries, by Bernoulli's law

$$\Delta p_j - \Delta p_a = p_2 - p_1 = \rho \frac{V_1^2 - V_2^2}{2} = \rho \frac{V_1 + V_2}{2} (V_1 - V_2), \tag{12}$$

since the total pressure in the basic flow is constant. To a first approximation $(V_1 - V_2 \ll V)$,

$$\Delta p_j - \Delta p_a = \rho V \epsilon, \tag{13}$$

where $\epsilon = V_1 - V_2$. Thus a discontinuity in the velocity field corresponds to the pressure discontinuity supported by the jet. With respect to the action of the thin jet on the basic flow the jet can be thought of as a discontinuity surface of free transverse vortices with the vorticity (circulation per unit length) $\epsilon$. Because the static pressure is continuous through the jet boundaries the excess of total pressure in the jet is, according to Bernoulli's law

$$p_{tj} - p_{ta} = \Delta p_t = \rho \frac{(V+v)^2 - v^2}{2}. \tag{14}$$

Hence, by eq. (11),

$$\Delta p_j - \Delta p_a = 2\Delta p_t \frac{h}{r} \tag{15}$$

and by eq. (13)

$$\epsilon = 2 \frac{\Delta p_t}{\rho V} \frac{h}{r}. \tag{16}$$

The vorticity $\epsilon$ must vanish at infinity where the jet is completely turned in the direction of undisturbed velocity and where the radius of curvature has become infinite.

Now let $x$ and $y_j$ denote the coordinates of a point of the vortex sheet $\epsilon$ which substitutes for the jet and assume that the slope $dy_j/dx$ of the sheet is small and of first order everywhere. In this order of approximation

$$\frac{1}{r} = \frac{d^2 y_j}{dx^2}$$

and

$$\epsilon = 2 \frac{\Delta p_t}{\rho V} h \frac{d^2 y_j}{dx^2} \tag{17}$$

The slope of the vortex sheet behind the airfoil $(x \geqslant c)$ is determined by Biot-Savart's law

$$\frac{dy_j(x)}{dx} = \frac{w_j(x)}{V} = -\frac{1}{2\pi V} \left( \int_0^c \frac{\gamma(x')}{x-x'} dx' + \int_c^\infty \frac{\epsilon(x')}{x-x'} dx' \right)$$

$$= \frac{-1}{2\pi V} \left( \int_0^c \frac{\gamma(x')}{x-x'} dx' + 2 \frac{\Delta p_t}{\rho V} h \int_c^\infty \frac{d^2 y_j(x')}{dx'^2} \frac{dx'}{x-x'} \right) \tag{18}$$

where $w_j(x)$ denotes the local downwash velocity, $c$ the chord length of the airfoil and $\gamma(x)$ the lifting vorticity within the airfoil $(0 < x < c)$. At the airfoil itself the flow must

follow the slope $dy_L/dx$ of the lifting surface (airfoil mean line):

$$\frac{dy_L(x)}{dx} = -\frac{1}{2\pi V}\left(\int_0^c \frac{\gamma(x')}{x-x'}\, dx' + 2\frac{\Delta p_t}{\rho V}\, h \int_c^\infty \frac{d^2 y_j(x')}{dx'^2}\, \frac{dx'}{x-x'}\right) \quad (19)$$

The compound of the integral equations 18 and 19 determines the unknown distributions of lifting vorticity $\gamma$ over the chord of the airfoil and of free vorticity $\epsilon$ downstream of the airfoil. Since the turning of the jet by the basic flow generally starts right at the trailing edge, the vorticity will not vanish there ($\gamma = \epsilon$), except in the case of a symmetrical airfoil with a jet blowing in the direction of the axis of symmetry at zero incidence, when no lift occurs at all.

The Kutta-Joukowski theorem can be applied in two ways: **first**, by integration of the local Kutta-Joukowski forces along the chord

$$\frac{L}{b} = \int_0^c \rho(V+\Delta V)\gamma\, dx, \quad (20)$$

where $\Delta V$ denotes the x-component of the disturbance velocity induced by the transverse vortices behind the airfoil. These are situated below the level of the trailing edge and have the same sense of rotation as the lifting vortices.
**Second**, by including the integral of the free transversal vorticity in the effective circulation

$$\frac{L}{b} = \rho V \Gamma_\infty = \rho V \left(\int_0^c \gamma\, dx + \int_c^\infty \epsilon\, dx\right). \quad (21)$$

The expressions 20 and 21 must turn out to be equal, but the second one will more easily be computed. Since the second integral must be finite the vorticity $\epsilon$ must decay faster than $1/x$ with x tending to infinity.

The action of the jet on the airfoil is to increase the relative velocity by an amount $\Delta U$, to increase the angle of attack by an upwash $\Delta\alpha$ and to protect the lift vorticity from dropping to zero at the trailing edge. Both of the additional quantities $\Delta V$ and $\Delta\alpha$ increase toward the trailing edge.

# SUMMARY

The physical problem of the lift of a blowing wing in a parallel stream is discussed and the integral equations for the two-dimensional case are derived. In this case no downwash is left at infinity downstream of the wing and the initial downwash momentum of the jet is completely used to build up an additional circulation. Essentially the jets act as an extension of the wing chord, raising the lifting vorticity at the trailing edge to a finite positive value.

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